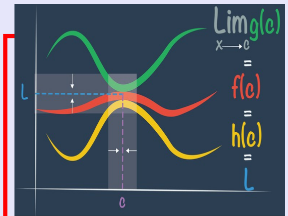
Math 261

Fall 2022

Lecture 8



Class QZ 2 Portrait Style

1) Find $\lim\limits_{x \to 3} \dfrac{x^2-9}{x^2+2x-3} = \dfrac{3^2-9}{3^2+2(3)-3} = \dfrac{0}{12} = \boxed{0}$ ✓

2) Find $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x-4} = \dfrac{\sqrt{4}-2}{4-4} = \dfrac{0}{0}$ I.F.

$$\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x \to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)} = \lim_{x \to 4} \frac{x-4}{(x-4)(\sqrt{x}+2)}$$

✓

$$= \lim_{x \to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

Properties of limit:

$\lim_{x \to a} f(x)$ & $\lim_{x \to a} g(x)$ exist.

1) $\lim_{x \to a} C = C$

2) $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

3) $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

4) $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\lim_{x \to a} g(x) \neq 0$

6) $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$

7) $\lim_{x \to a} x^n = a^n$ n is positive integer

8) $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ if n is even, $a > 0$

Find $\lim_{x \to a} f(x)$ & $\lim_{x \to a} g(x)$ given

$$\begin{cases} \lim_{x \to a} [f(x) + g(x)] = 10 \\ \lim_{x \to a} [f(x) - g(x)] = 2 \end{cases} \Rightarrow \begin{cases} \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = 10 \\ \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = 2 \end{cases}$$

$2 \lim_{x \to a} f(x) = 12$

$\boxed{\lim_{x \to a} f(x) = 6}$

Plug it in one of equs:

$\lim_{x \to a} f(x) + \lim_{x \to a} g(x) = 10$

$6 + \lim_{x \to a} g(x) = 10$

$\boxed{\lim_{x \to a} g(x) = 4}$

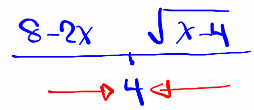
Suppose $\lim_{x \to 4} f(x) = 10$

Find $\lim_{x \to 4} \dfrac{x^2+4}{x f(x)} = \dfrac{\lim_{x \to 4}(x^2+4)}{\lim_{x \to 4} x f(x)}$

$= \dfrac{\lim_{x \to 4} x^2 + \lim_{x \to 4} 4}{\lim_{x \to 4} x \cdot \lim_{x \to 4} f(x)}$

$= \dfrac{4^2+4}{4 \cdot 10} = \dfrac{20}{40} = \boxed{\dfrac{1}{2}}$

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8-2x & \text{if } x < 4 \\ 5 & \text{if } x = 4 \end{cases}$$



$\lim_{x \to 4^-} f(x) = 8-2(4) = \boxed{0}$

$\lim_{x \to 4^+} f(x) = \sqrt{4-4} = \boxed{0}$

$\lim_{x \to 4} f(x) = \boxed{0}$

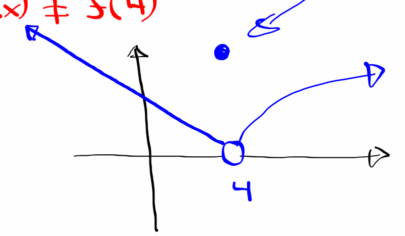
$f(4) = \boxed{5}$

Is $f(x)$ cont. at $x=4$? Explain

NO, $\lim_{x \to 4} f(x) \neq f(4)$

$f(4)=5$

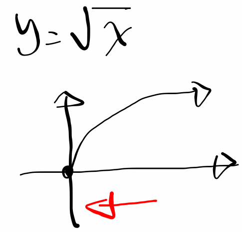
Graph $f(x)$



Range ⇒ $(0, \infty)$

Interval notation

Prove $\lim_{x\to 0^+} \sqrt{x}\left[1 + \sin^2 \frac{2\pi}{x}\right] = 0$



Is it OK if

$$\lim_{x\to 0^+} \sqrt{x} \cdot \lim_{x\to 0^+} \left[1 + \sin^2 \frac{2\pi}{x}\right] =$$

has to be defined.

$$0 \cdot \text{whatever} = 0$$

let's take a look at

$$\lim_{x\to 0^+} \left(1 + \sin^2 \frac{2\pi}{x}\right) = \lim_{x\to 0^+} 1 + \lim_{x\to 0^+} \sin^2 \frac{2\pi}{x}$$

does this limit exist?

We are working with $\lim_{x\to 0^+} \sin^2 \frac{2\pi}{x}$

using Squeeze thrm

$-1 \le \sin A \le 1 \Rightarrow 0 \le \sin^2 A \le 1$

$$0 \le \sin^2 \frac{2\pi}{x} \le 1$$

Recall

$$\lim_{x\to 0^+} \sqrt{x}\left[1 + \sin^2 \frac{2\pi}{x}\right] = \cancel{\lim_{x\to 0^+} \sqrt{x}} \to 0 + \lim_{x\to 0} \sqrt{x} \sin^2 \frac{2\pi}{x}$$

$$0 \le \sin^2 \frac{2\pi}{x} \le 1$$

$$0 \cdot \sqrt{x} \le \sqrt{x} \sin^2 \frac{2\pi}{x} \le 1 \cdot \sqrt{x}$$

$$0 \le \sqrt{x} \sin^2 \frac{2\pi}{x} \le \sqrt{x}$$

$\lim_{x\to 0^+} 0 = 0$ $\qquad$ $\lim_{x\to 0^+} \sqrt{x} = 0$

By S.T.

$$\lim_{x\to 0^+} \sqrt{x} \sin^2 \frac{2\pi}{x} = \boxed{0}$$

For $\varepsilon > 0$, Find a $\delta > 0$ Such that $\lim_{x \to 2}(4x-5) = 3$

Verify the limit

$\lim_{x \to 2}(4x-5) = 4(2)-5 = 3$ ✓

$f(x) = 4x-5$, $a = 2$, $L = 3$

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|4x-5-3| < \varepsilon$ " $|x-2| < \delta$

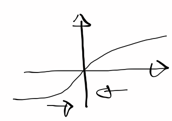
$|4x-8| < \varepsilon$ " $|x-2| < \delta$

$4|x-2| < \varepsilon$ " $|x-2| < \delta$

$|x-2| < \frac{\varepsilon}{4}$ " $|x-2| < \delta$

Pick $\delta = \frac{\varepsilon}{4}$

For $\varepsilon > 0$, find $\delta > 0$ Such that $\lim_{x \to 0}\sqrt[3]{x} = 0$



Verify the limit

$\lim_{x \to 0}\sqrt[3]{x} = \sqrt[3]{0} = 0$ ✓

$f(x) = \sqrt[3]{x}$, $a = 0$, $L = 0$

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|\sqrt[3]{x} - 0| < \varepsilon$ " $|x-0| < \delta$

$|\sqrt[3]{x}| < \varepsilon$ $|x| < \delta$

Cube both Sides

$|\sqrt[3]{x}|^3 < \varepsilon^3$ " $|x| < \delta$

$|x| < \varepsilon^3$

Pick $\delta = \varepsilon^3$

For $\varepsilon>0$, find a $\delta>0$ such that

$\lim_{x\to 1}(x^2+4x-2)=3.$

verify $\lim_{x\to 1}(x^2+4x-2)=1^2+4(1)-2=3$ ✓

$f(x)=x^2+4x-2$, $a=1$, $L=3$

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^2+4x-2-3|<\varepsilon$ " $|x-1|<\delta$

$|x^2+4x-5|<\varepsilon$

$|(x+5)(x-1)|<\varepsilon$ Keep

$|x+5|\,|x-1|<\varepsilon$

Bound

Suppose we want $\delta\le 1 \Rightarrow |x-1|\le 1$

$-1\le x-1\le 1$

we want $x+5$

Add 6

$-1+6\le x-1+6\le 1+6$

$-7\le 5\le x+5\le 7$

$-7\le x+5\le 7$

$|x+5|\le 7$

$7|x-1|<\varepsilon$

$|x-1|<\frac{\varepsilon}{7}$ Pick $\delta=\min\{1,\frac{\varepsilon}{7}\}$

$\varepsilon=1 \to \delta=\frac{1}{7}$

$\varepsilon=5 \to \delta=\frac{5}{7}$

$\varepsilon=7 \to \delta=1$

$\varepsilon\to 10 \to \delta=\min\{1,\frac{10}{7}\}=1$